Discovering Conservation Laws using Optimal Transport and Manifold Learning

THE UNIVERSITY OF CHICAGO DATA SCIENCE INSTITUTE

UChicago AI+Science Summer School
August 8–12, 2022

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Overview

Conservation laws are key theoretical and practical tools for understanding, characterizing, and modeling nonlinear dynamical systems.

Current approaches for discovering conservation laws often depend on detailed dynamical information, such as the equation of motion or fine-grained time measurements, with many recent proposals also relying on black box parametric deep learning methods (Wetzel et al. 2020, Ha & Jeong 2021).

We instead reformulate this task as a manifold learning problem and propose a non-parametric approach.

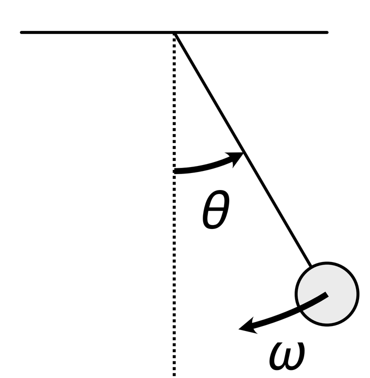
Using tools from **optimal transport theory** and **manifold learning**, our proposed method provides a **direct geometric approach to identifying conservation laws** that is both **robust** and **interpretable** without requiring an explicit model of the system nor accurate time information.

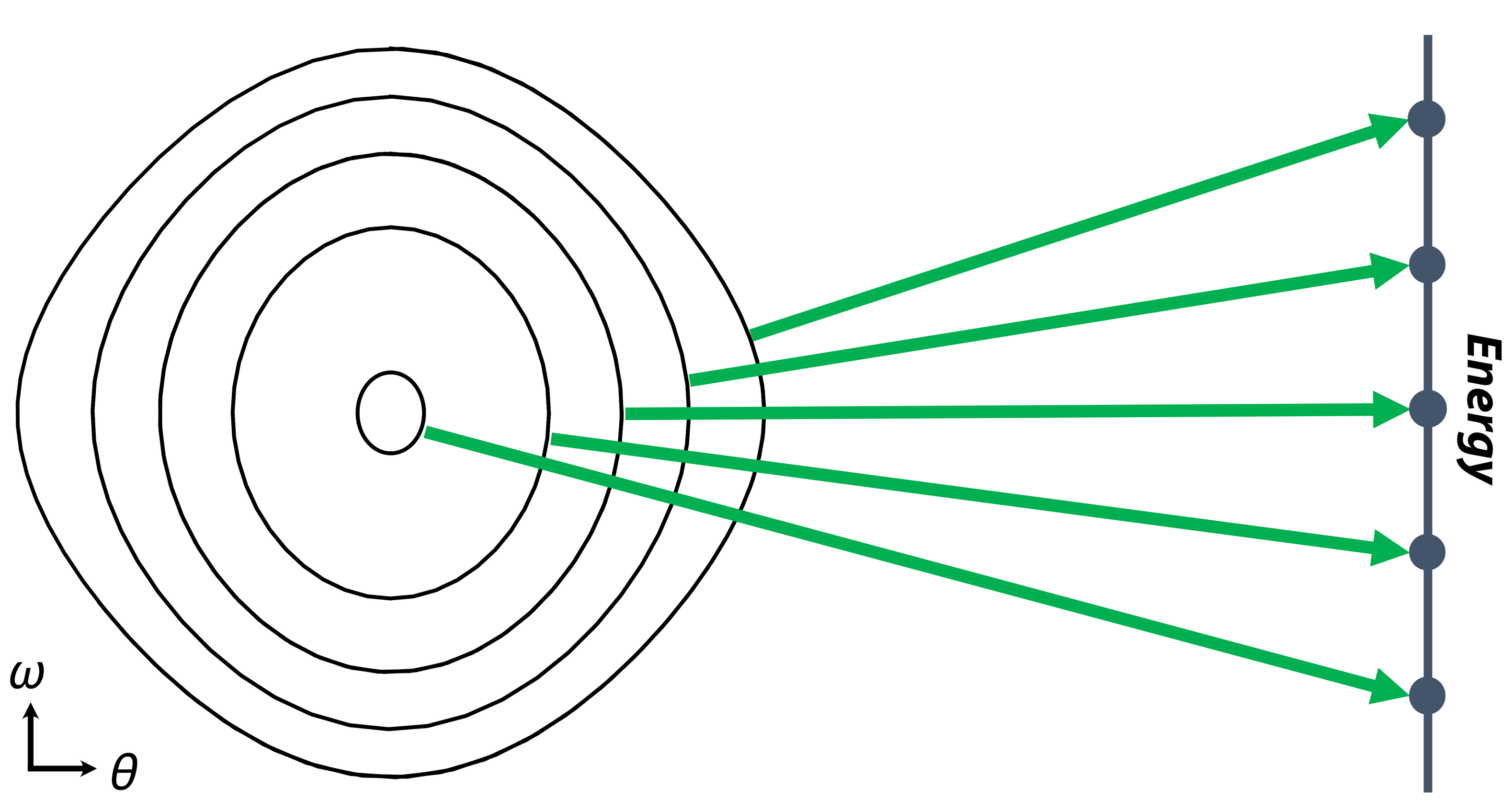
Formulation & Approach

Observation: Isosurfaces of conserved quantities partition phase space. Each trajectory is restricted to an isosurface.

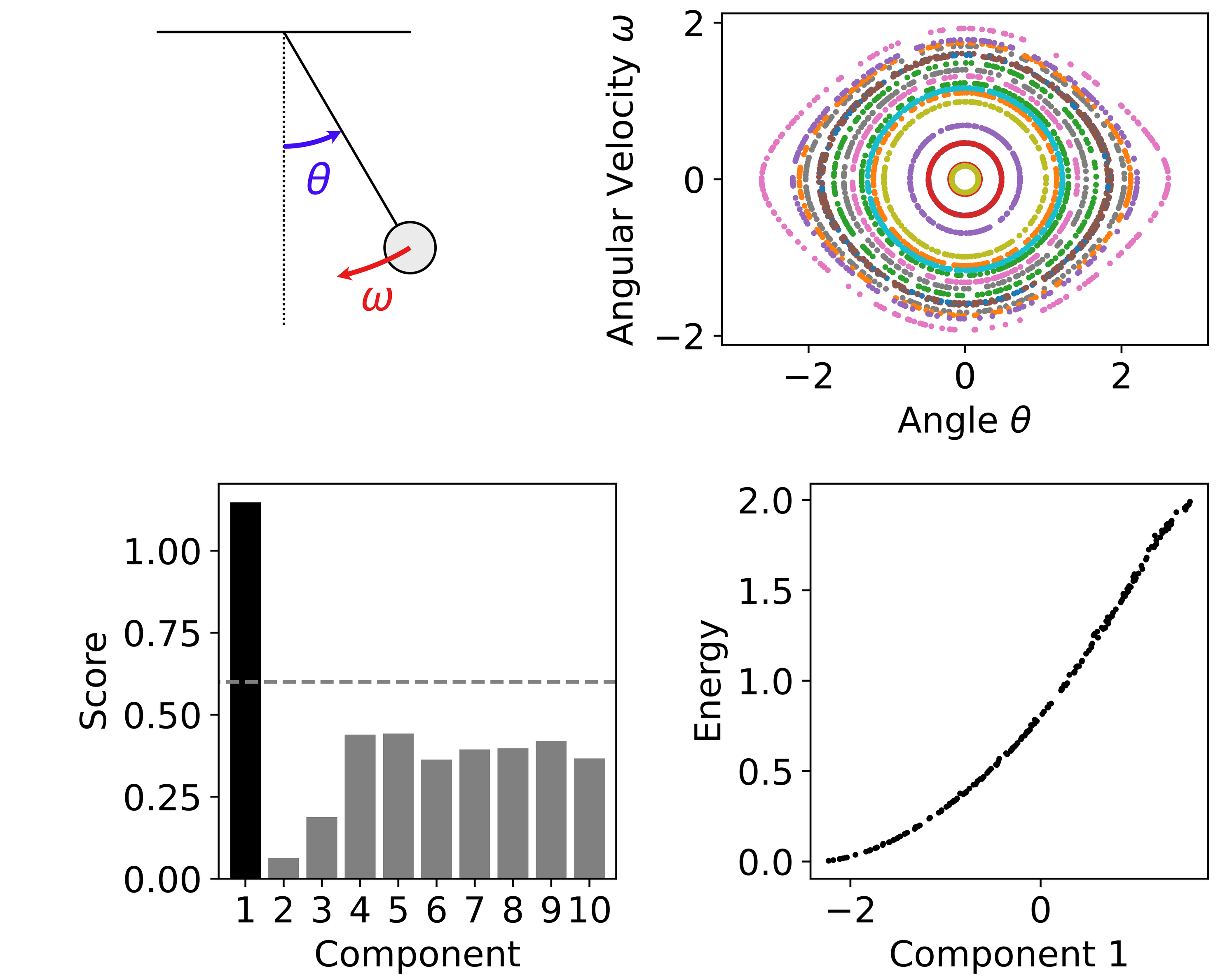
Goal: Obtain explicit parameterization of all varying conserved quantities in the data.

Idea: Parameterize changes in shape of isosurfaces via manifold learning (diffusion maps using the Wasserstein metric).

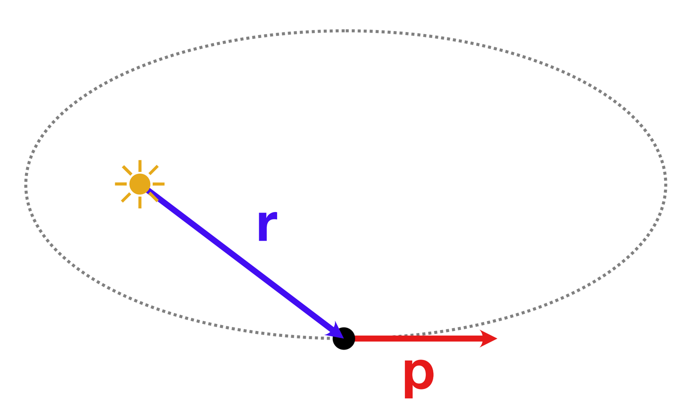




Simple Pendulum



Planar Gravitational Dynamics



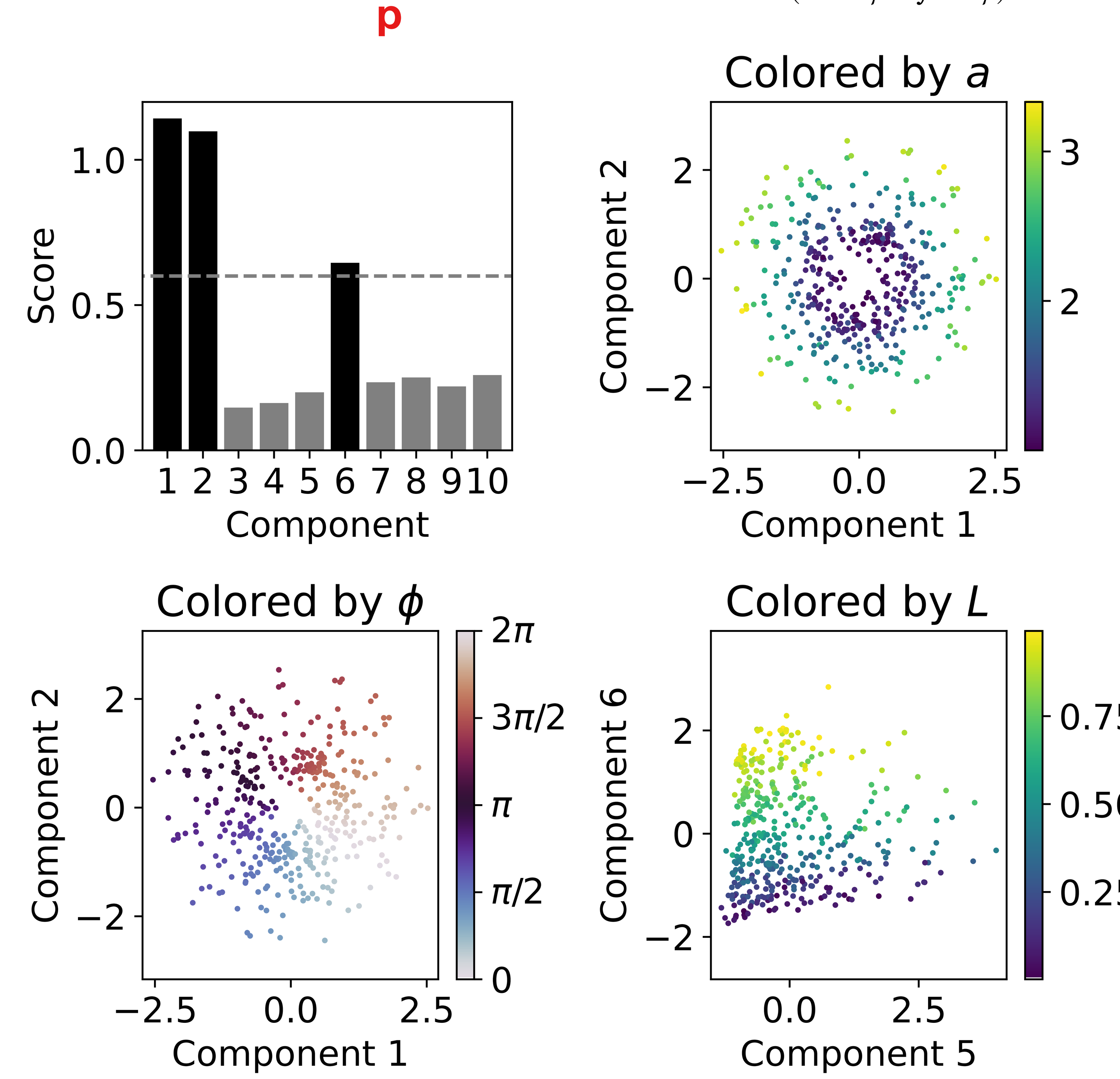
$$E = \frac{\mathbf{p}^2}{2} - \frac{1}{|\mathbf{r}|} = -\frac{1}{2a}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = L\hat{\mathbf{z}}$$

$$\mathbf{A} = \mathbf{p} \times \mathbf{L} - \hat{\mathbf{r}}$$

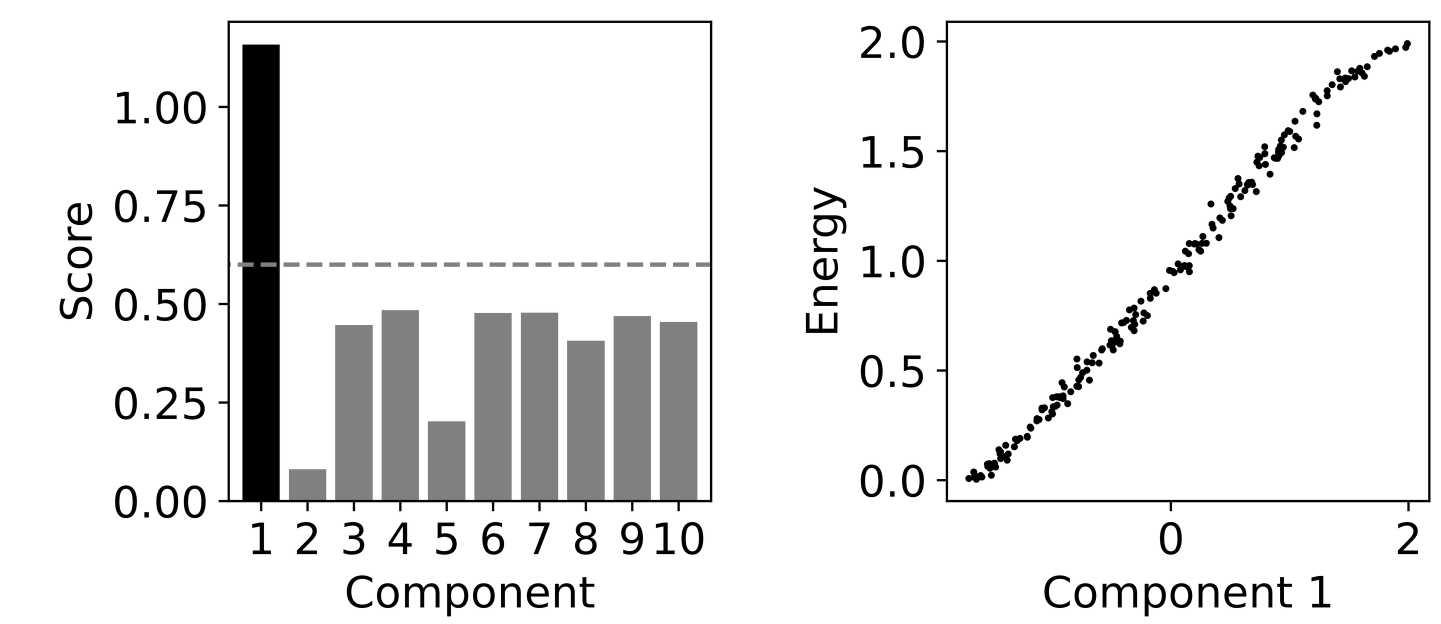
$$= \sqrt{1 + 2EL^2}$$

$$(\hat{\mathbf{x}} \cos\phi + \hat{\mathbf{y}} \sin\phi)$$



Simple Pendulum with Noise

Added Gaussian noise with σ = 0.5



Double Pendulum