Discovering Dynamical Parameters by Interpreting Echo State Networks

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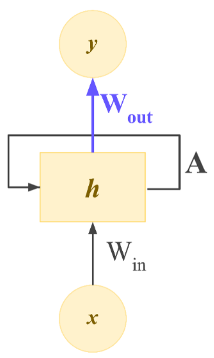
Background: Echo State Networks

ESN architecture:
- Randomly initialized RNN-like architecture
- Sparse reservoir **A**, input weights $\mathbf{W}_{\text{in}}$ both fixed
- Only output/readout layer $\mathbf{W}_{\text{out}}$ is trained (via *linear regression*)



Strengths:
- High prediction performance
- Efficient and stable learning of chaotic dynamics
- Computationally inexpensive

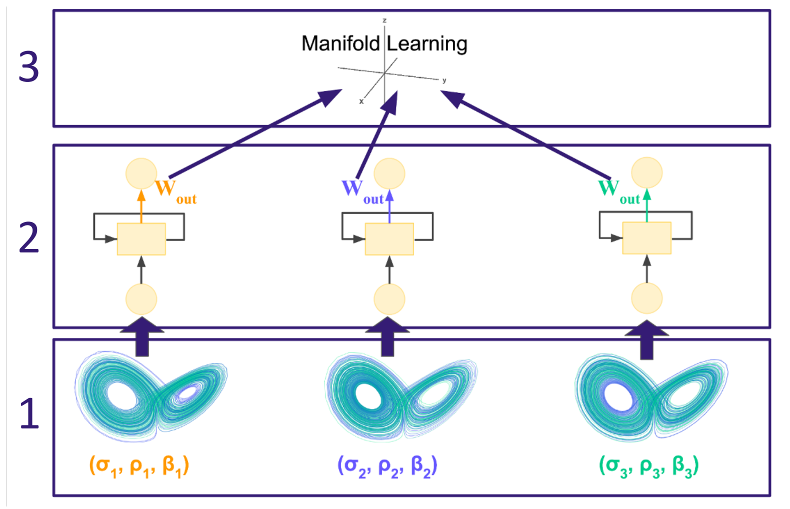
ESN applications in forecasting:
- Improve training stability on chaotic data
- Amplify forecasting performance

New application and approach: *Interpret ESNs to discover varying dynamical parameters in trajectory data.*

Question: *Are dynamical parameters embedded in the readout layer?*

Interpreting ESNs Using Manifold Learning

1. **Collect trajectory data from dynamical system** with uncontrolled or varying dynamical parameters. We use data from the Lorenz system.
2. **Train an ESN on each trajectory**, where each ESN shares the same reservoir so that corresponding readout weights are comparable.
3. **Perform manifold learning** to identify components of the readout layers that correspond to dynamical parameters which differ between trajectories in the data.

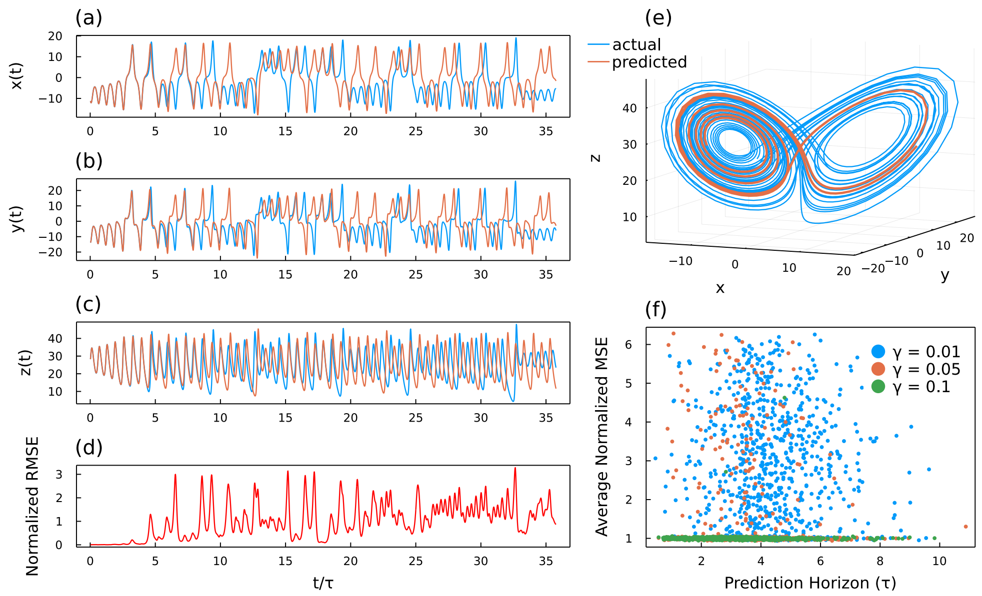


Optimizing Prediction Performance

Lorenz system with 3 varying dynamical parameters σ, ρ, β:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y-x), \quad \frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y, \quad \frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z$$

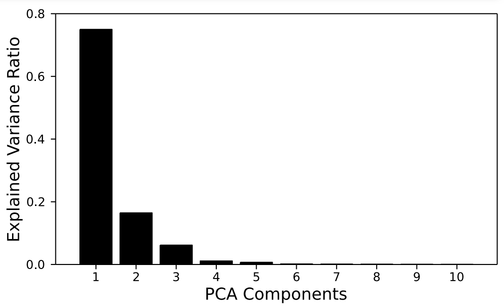
Accurate ESN models of trajectory data necessary for extracting high quality representations of the dynamics from ESN readout layers:
- Prediction performance **(a–e)**:
  Good short-term forecasting (~4 Lyapunov times τ)
- Effect of ridge regression regularization **(f)**:
  Higher regularization → improved long-term stability



Applying Manifold Learning: PCA

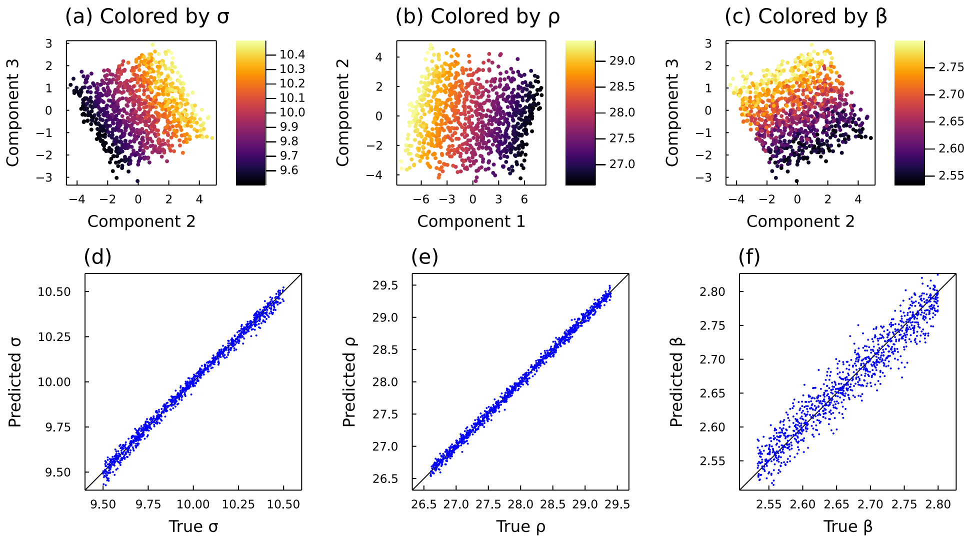
1. **Apply PCA** to flattened ESN readout layer weights.
2. **Identify relevant PCA components** using explained variance ratio.

(total variance threshold: 0.95)



Extracting Dynamical Parameters

PCA components form embedding of dynamical parameters:
- PCA projections **(a–c)**: Relevant components correspond to the varying dynamical parameters σ, ρ, β.
- Fitted parameters vs. ground truth **(d–f)**: Linear fits of the extracted embedding to σ, ρ, β with R^2 = 0.993, 0.996, 0.925, respectively.



Conclusions

We have demonstrated on the Lorenz system that…
- ESN readout layers contain encoding of the dynamical parameters.
- System parameters can be extracted through manifold learning.

Potential applications for ESNs in interpretable modeling:
- Provides an alternative, computationally inexpensive method for unsupervised identification of dynamics.
- Easily applied to any existing ESN prediction models.
- Acts as a generic dynamics encoder for nonlinear dynamics.

Future work:
- Test on high-dimensional data, e.g. from PDEs.
- Adapt for parameter extraction with only partial state information.
- Develop better theoretical understanding of how ESN dynamics are encoded in the readout layer.

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