Discovering Sparse Interpretable Dynamics from Partial Observations

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Overview

Identifying the governing equations of a nonlinear dynamical system is key to both understanding the physical features of the system and constructing an accurate model of the dynamics that generalizes well beyond the available data.

In many cases, this problem is further compounded by a lack of available data and only **partial observations of the system state**, e.g. forecasting fluid flow driven by unknown sources or predicting optical signal propagation without phase measurements.

While many deep learning methods exist for learning models from partial observations, very few approaches provide any interpretability. **Sparse symbolic system identification methods** are both highly **interpretable** and provide an excellent **physics-informed inductive bias** for dynamical systems found in nature, but cannot handle partially observed systems.

To accomplish **interpretable partially observed system identification**, we propose a machine learning framework that combines an **encoder for state reconstruction** with a **sparse symbolic model**. Our tests show that this method can successfully **reconstruct the full system state** and **identify the equations of motion** governing the underlying dynamics for a variety of ODE and PDE systems.

Problem Formulation

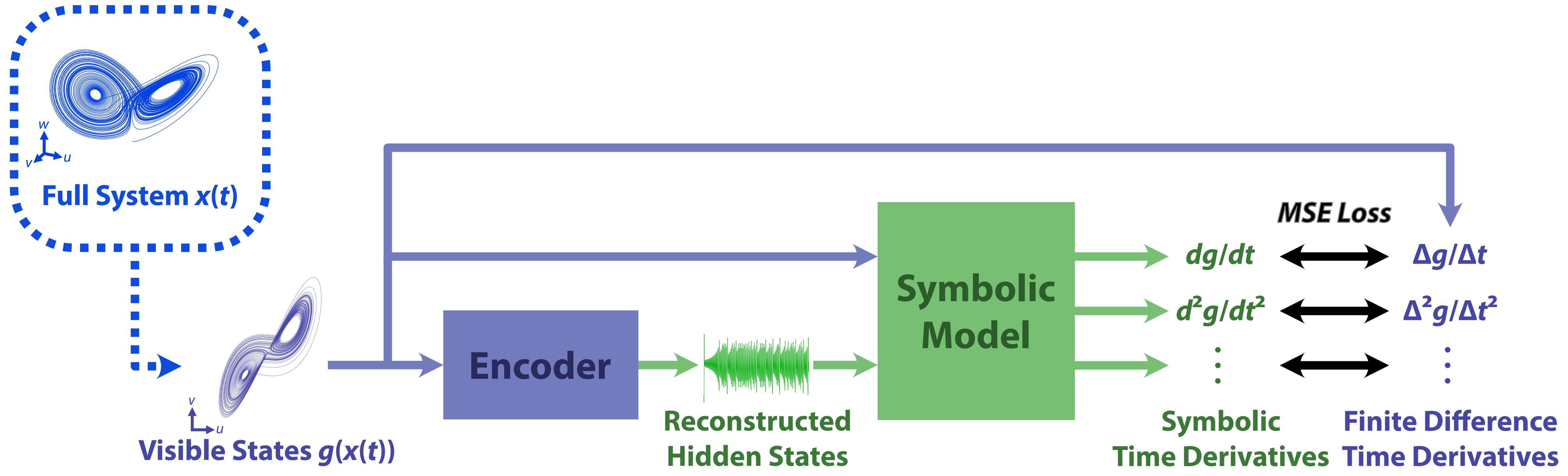
We only have data from an **observed visible state**, which is related to the full system state by a known **projection function**. A known **aggregation function** can reconstruct the full system state from the visible state and an **unknown hidden state**.

$$\mathbf{x}_v = \mathbf{g}(\mathbf{x})$$

$$\mathbf{x} = \mathbf{a}(\mathbf{x}_v, \mathbf{x}_h)$$

Goal: *Identify the system dynamics* $\dfrac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{F}(\mathbf{x})$.

Machine Learning Framework



Our proposed machine learning framework consists of:

(1) a **deep learning encoder**, for reconstructing hidden states from **partial observations**; and

(2) a **sparse symbolic model**, for learning the explicit **symbolic governing equations**.

The entire architecture is trained end-to-end by **matching the higher-order symbolic time derivatives** of the sparse symbolic model with finite difference estimates from the data.

ODE Identification Experiments

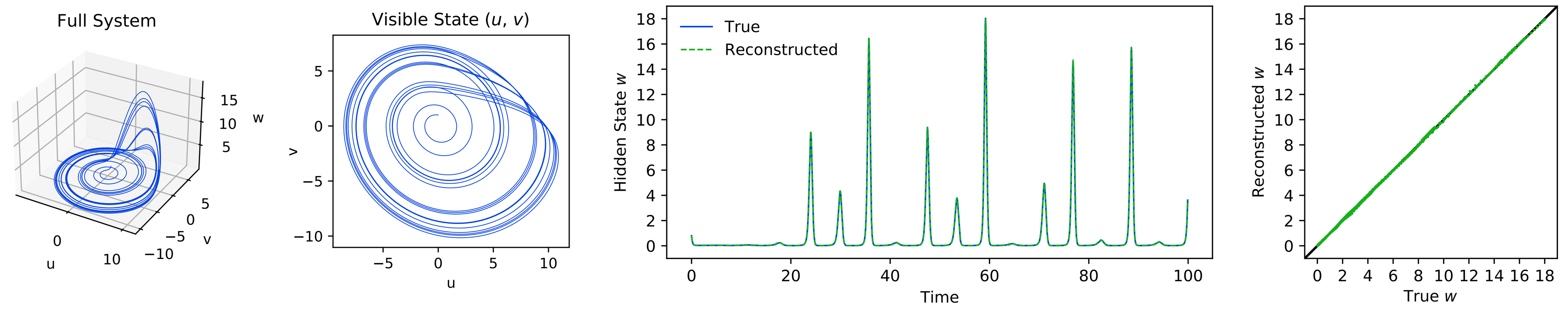
Rössler System

True Governing Equations

$$\frac{du}{dt} = -v - w,\quad \frac{dv}{dt} = u + 0.2v,\quad \frac{dw}{dt} = 0.2 - 5.7w + uw$$

Reconstructed Governing Equations

$$\frac{du}{dt} = -1.00v - 1.00w,\quad \frac{dv}{dt} = 1.00u + 0.20v,\quad \frac{dw}{dt} = 0.20 - 5.71w + 1.00uw$$



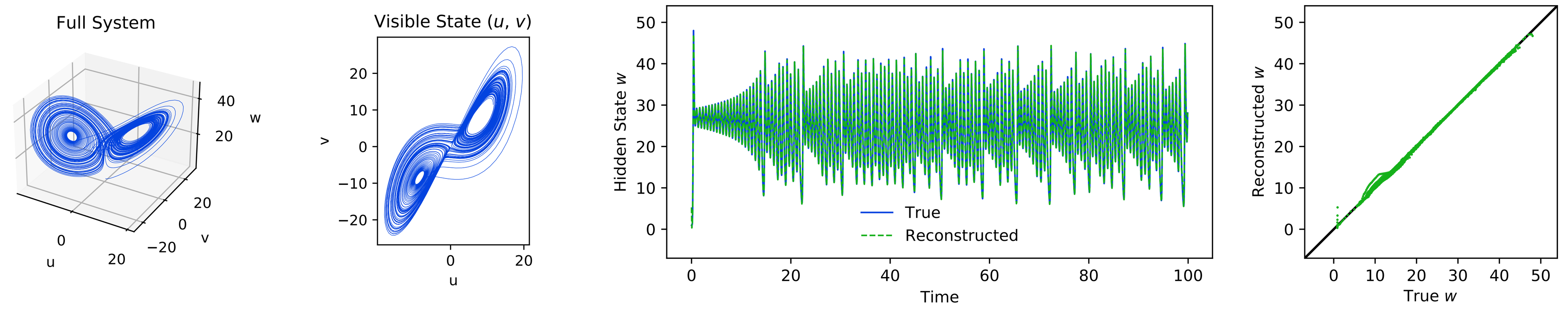
Lorenz System

True Governing Equations

$$\frac{du}{dt} = -10u + 10v,\quad \frac{dv}{dt} = 28u - v - uw,\quad \frac{dw}{dt} = -\frac{8}{3}w + uv$$

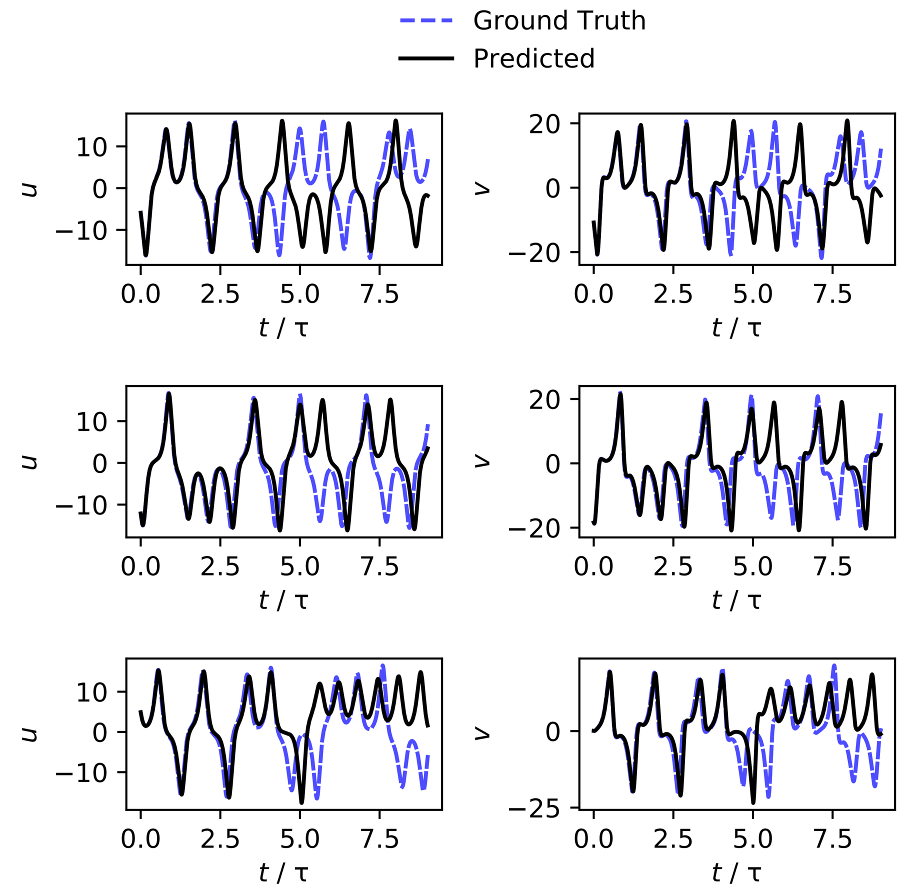
Reconstructed Governing Equations

$$\frac{du}{dt} = -9.99u + 9.99v,\quad \frac{dv}{dt} = 27.74u - 0.91v - 0.99uw,\quad \frac{dw}{dt} = 0.03 - 2.72w + 0.99uv$$

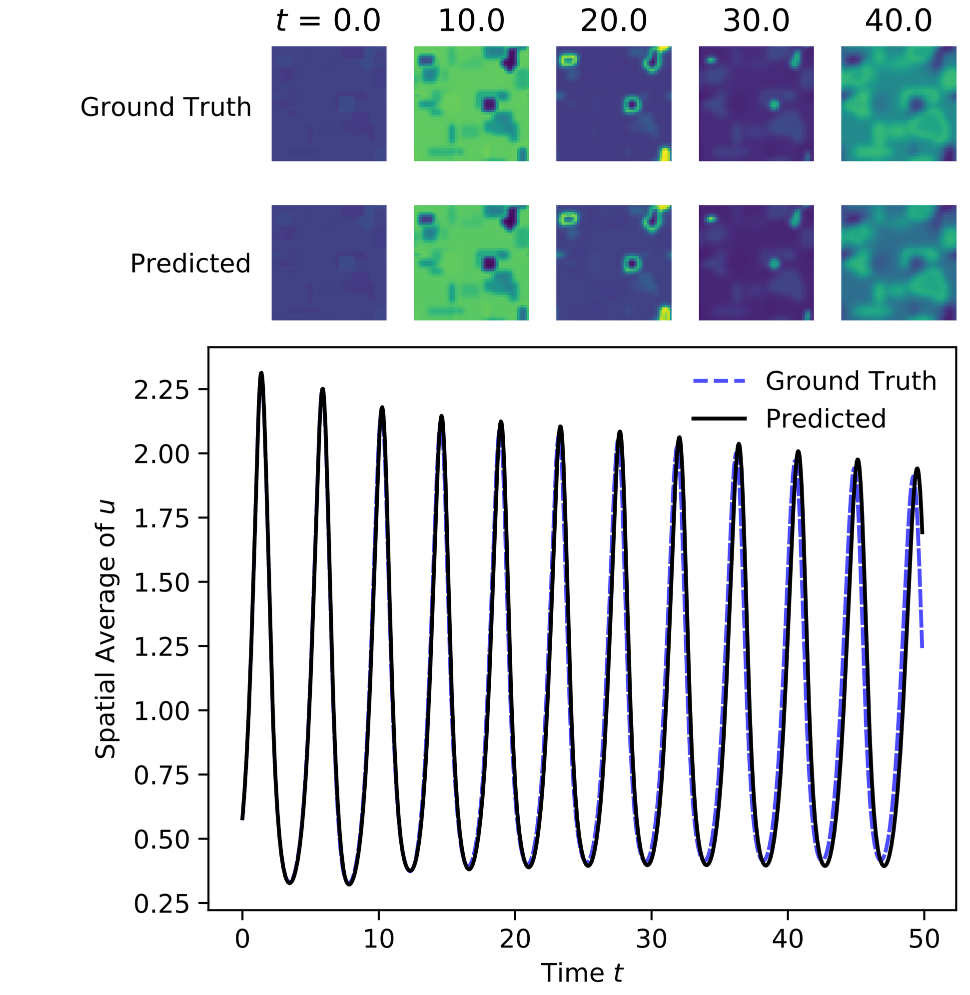


Prediction Examples

Lorenz System



Diffusive Lotka–Volterra



PDE Identification Experiments

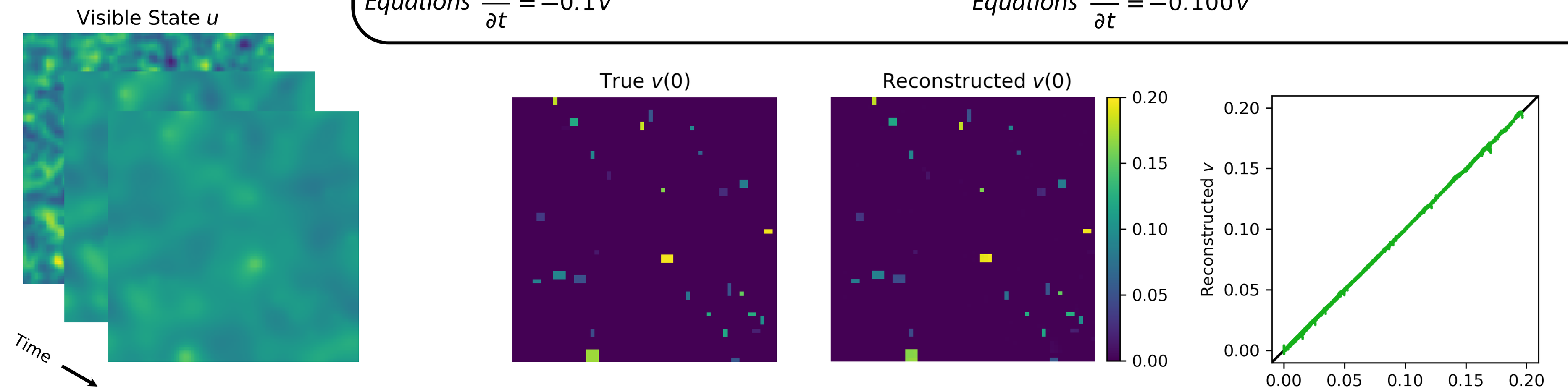
Diffusion with Source

True Governing Equations

$$\frac{\partial u}{\partial t} = 0.2\,\partial_{xx}u + 0.2\,\partial_{yy}u + v,\quad \frac{\partial v}{\partial t} = -0.1v$$

Reconstructed Governing Equations

$$\frac{\partial u}{\partial t} = 0.200\,\partial_{xx}u + 0.200\,\partial_{yy}u + 0.999v,\quad \frac{\partial v}{\partial t} = -0.100v$$



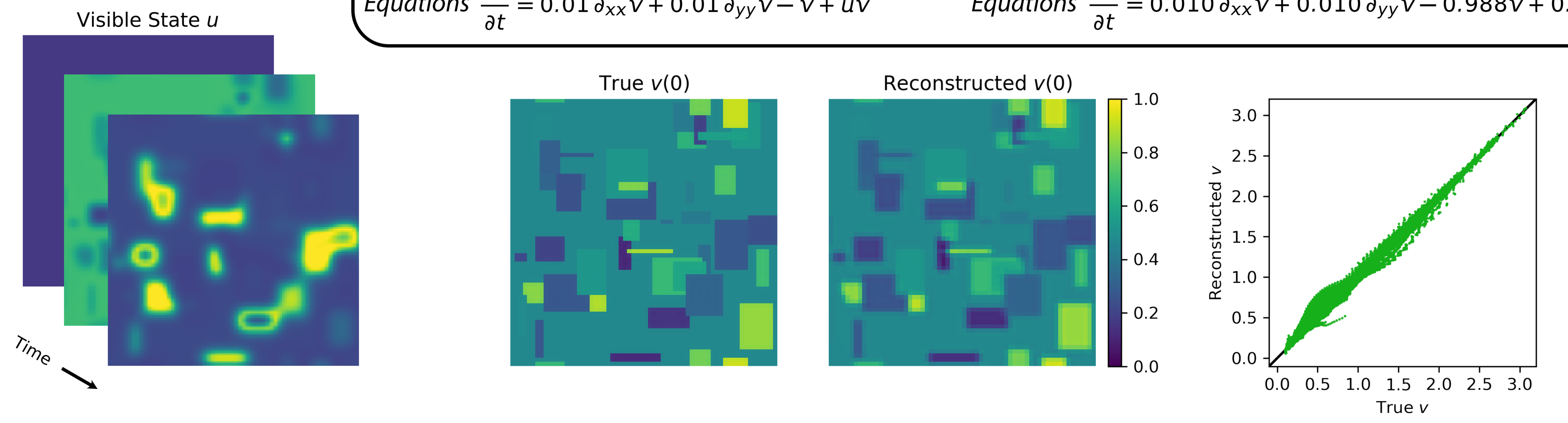
Diffusive Lotka–Volterra

True Governing Equations

$$\frac{\partial u}{\partial t} = 0.05\,\partial_{xx}u + 0.05\,\partial_{yy}u + \frac{7}{3}u - \frac{8}{3}uv,\quad \frac{\partial v}{\partial t} = 0.01\,\partial_{xx}v + 0.01\,\partial_{yy}v - v + uv$$

Reconstructed Governing Equations

$$\frac{\partial u}{\partial t} = 0.055\,\partial_{xx}u + 0.053\,\partial_{yy}u + 2.333u - 2.667uv,\quad \frac{\partial v}{\partial t} = 0.010\,\partial_{xx}v + 0.010\,\partial_{yy}v - 0.988v + 0.991uv$$



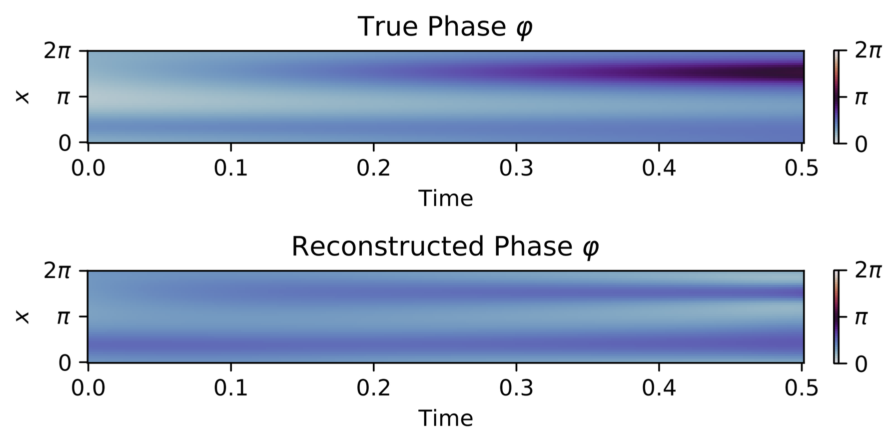
Phase Reconstruction Experiment

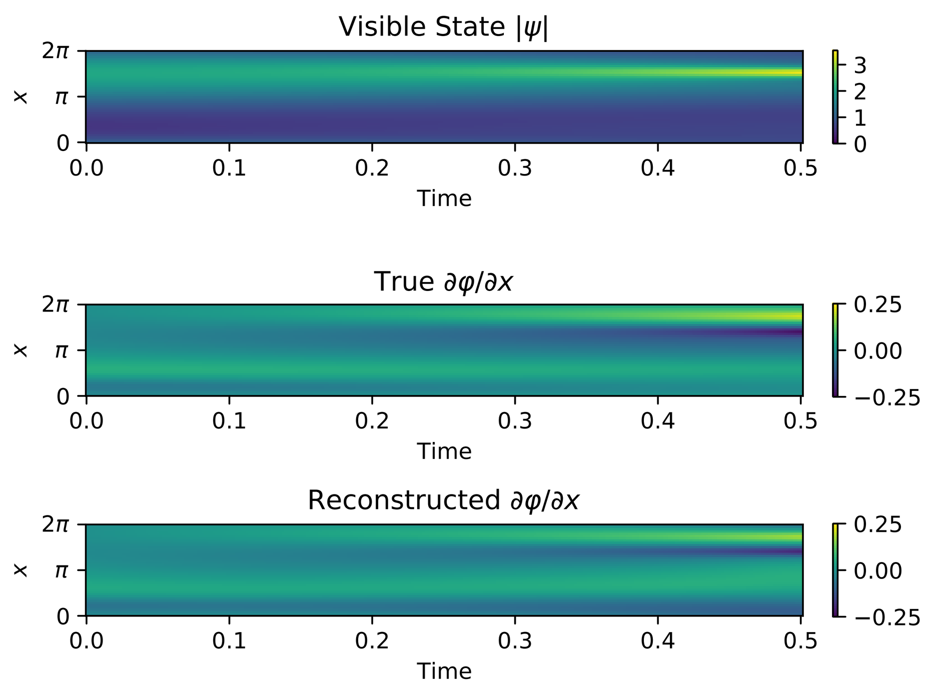
True Governing Equations

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\partial_{xx}\psi - |\psi|^2\psi$$

Reconstructed Governing Equations

$$i\frac{\partial\psi}{\partial t} = -0.52\,\partial_{xx}\psi - 1.07\,|\psi|^2\psi$$





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